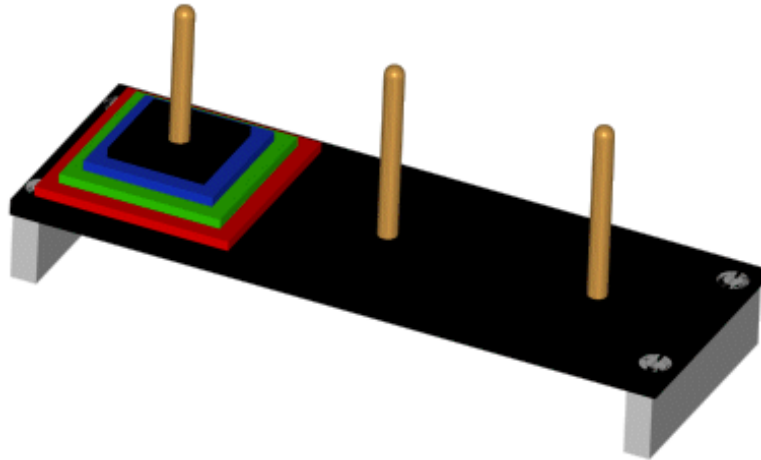


The Towers of Hanoi



Towers of Hanoi is a mathematical game or puzzle. It consists of three pegs, and a number of disks of different sizes which can slide onto any peg. The puzzle starts with the disks neatly stacked in order of size on one peg, smallest at the top, thus making a conical shape.

The objective of the game is to move the entire stack to another peg, obeying the following rules:

- ❖ Only one disk may be moved at a time.
- ❖ Each move consists of taking the upper disk from one of the pegs and sliding it onto another peg, on top of the other disks that may already be present on that peg.
- ❖ No disk may be placed on top of a smaller disk.

The puzzle was invented by the French mathematician Édouard Lucas in 1883. There is a legend about an Indian temple which contains a large room with three time-worn posts in it surrounded by 64 golden disks. The priests of Brahma, acting out the command of an ancient prophecy, have been moving these disks, in accordance with the rules of the puzzle. According to the legend, when the last move of the puzzle is completed, the world will end. The puzzle is therefore also known as the Tower of Brahma puzzle. It is not clear whether Lucas invented this legend or was inspired by it.

If the legend were true and if the priests were able to move disks at a rate of 1 per second, using the smallest number of moves, it would take them 2^{64} seconds or roughly 584.554 billion years. The universe is currently about 13.7 billion years old.

You can add more disks, thus making the puzzle more difficult. If you have the time you can add disks of sizes 55 x 55, 45 x 45, 35 x 35, and 25 x 25.

With 4 disks the minimum number of moves is 15. With 8 disks the minimum number of moves is 255.

$$2^4 - 1 = (2 \times 2 \times 2 \times 2) - 1 = 15$$

$$2^8 - 1 = (2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2) - 1 = 255$$

$$2^{64} - 1 = 18446744073709551615$$